Evaluate the integral using the indicated trigonometric substitution. Sketch and label the associated right triangle.

1) $\int \frac{1}{x^2\sqrt{x^2-9}}\,dx$ $\quad x=3\sec\theta$ $\quad \boxed{\frac{1}{9}\frac{\sqrt{x^2-9}}{x}+C}$

2) $\int x^3\sqrt{9-x^2}\,dx$ $\quad x=3\sin\theta$ $\quad \boxed{-\frac{1}{5}(x^2+6)(9-x)^{3/2}+C}$

3) $\int \frac{x^3}{\sqrt{x^2+9}}\,dx$ $\quad x=3\tan\theta$ $\quad \boxed{\frac{1}{3}(x^2-18)\sqrt{x^2+9}+C}$

4) $\int_0^{2\sqrt{3}} \frac{x^3}{\sqrt{16-x^2}}\,dx$ $\boxed{\frac{40}{3}}$

5) $\int_{\sqrt{2}}^{2} \frac{1}{t^3\sqrt{t^2-1}}\,dt$ $\boxed{\frac{\pi}{24}+\frac{\sqrt{3}}{8}-\frac{1}{4}}$

6) $\int_0^{2} x^3\sqrt{x^2+4}\,dx$ $\boxed{\frac{64}{15}\left(\sqrt{2}+1\right)}$

7) $\int \sqrt{1-4x^2}\,dx$

$$\boxed{\frac{1}{4}\left[\sin^{-1}(2x)+2x\sqrt{1-4x^2}\right]+C}$$

8) $\int \frac{1}{\sqrt{9x^2+6x-8}}\,dx$

$$\boxed{\frac{1}{3}\ln\left|3x+1+\sqrt{9x^2+6x-8}\right|+C}$$

9) $\int \frac{dx}{(x^2+2x+2)^2}$

$$\boxed{\frac{1}{2}\left[\tan^{-1}(x+1)+\frac{x+1}{x^2+2x+2}\right]+C}$$